

# Supplementary Materials for "Anomalous unidirectional excitation of high- $k$ hyperbolic modes using all-electric metasources"

Zhiwei Guo,<sup>1,2,\*</sup> Yang Long,<sup>1,3,†</sup> Haitao Jiang,<sup>1,2</sup> Jie Ren,<sup>1,3</sup> Hong Chen,<sup>1,2,3</sup>

<sup>1</sup> School of Physics Science and Engineering, Tongji University, Shanghai 200092, China

<sup>2</sup> Key Laboratory of Advanced Micro-structure Materials, MOE, Tongji University, Shanghai 200092, China

<sup>3</sup> Center for Phononics and Thermal Energy Science, China-EU Joint Lab On Nanophononics, Shanghai Key Laboratory of Special Artificial Microstructure Materials and Technology, Shanghai 200092, China

Corresponding authors:

\*Zhiwei Guo, [2014guozhiwei@tongji.edu.cn](mailto:2014guozhiwei@tongji.edu.cn), †Yang Long, [longyang\\_123@yeah.net](mailto:longyang_123@yeah.net).

## S1. Effective electromagnetic parameters of the circuit-based HMMs

In the circuit-based system, the relationship between the electric and magnetic fields can be easily mapped using the relationship between voltage and current. As a result, the electromagnetic response is equivalent to the circuit parameters. The structure factor of the TL is defined as  $g = Z_0 / \eta_{eff}$ , where  $Z_0$  and  $\eta_{eff}$  denote the characteristic impedance and effective wave impedance, respectively. For the TL model of the HMM in Fig. S1(a), the impedance and admittance of the circuit are represented by  $Z$  and  $Y$ , respectively. The direction of the magnetic field produced by the current can be determined from Ampere's law, as shown in Fig. S1(a). By mapping the circuit equation (telegraph equation) to Maxwell's equations, the relationship between circuit and electromagnetic parameters can be described by

$$\begin{aligned}\mu_{eff} \mu_0 / g &= Z / i\omega, \\ \varepsilon_{eff} \varepsilon_0 g &= Y / i\omega,\end{aligned}\tag{S1}$$

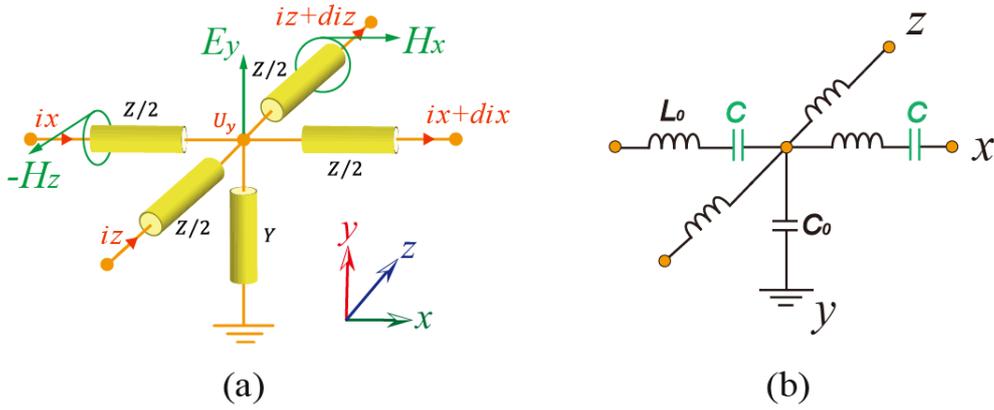
where  $\varepsilon_0$  and  $\mu_0$  are the vacuum permittivity and permeability, respectively.  $\omega$  is the angular frequency. The effective permittivity and permeability of the circuit system can be tuned using

the lumped elements in the circuit. Figure S1(b) shows a simple effective circuit model for a circuit-based HMM. In this circuit model, the admittance is  $Y = i\omega C_0$  and capacitors  $C$  are loaded in the  $x$  direction to realize anisotropic impedance

$$\begin{aligned} Z_x &= i\omega L_0, \\ Z_z &= i\omega L_0 + 1/i\omega C, \end{aligned} \quad (\text{S2})$$

where  $C_0$  and  $L_0$  denote the capacitance and inductance per unit length, respectively. Therefore, the effective electromagnetic parameters of the system are

$$\begin{aligned} \varepsilon &= 2C_0 g / \varepsilon_0, \quad \mu_x = \frac{L_0}{g \mu_0}, \\ \mu_z &= \frac{L_0}{g \mu_0} - \frac{1}{\omega^2 C d g \mu_0}. \end{aligned} \quad (\text{S3})$$



**Figure S1.** a) TL model of the circuit-based HMM. b) The corresponding anisotropic circuit model of the effective HMM.

## S2. Dispersion relation for TE polarization in HMM and normal waveguides

Since the tangential components of the electric and magnetic fields should be continuous at the boundary of the HMM and DPS media ( $\varepsilon_D \approx 3.63$ ,  $\mu_D = 1$ ), the dispersion relation of the lowest-order TE-polarized guided modes in HMM waveguide [Fig. 6(e)] can be deduced from the characteristic equation<sup>78,79</sup>

$$k_{zH} d = 2 \arctan \left[ \frac{\mu_x \text{Im}(k_{zD})}{\mu_D k_{zH}} \right], \quad (\text{S4})$$

where  $k_{zH} = \sqrt{\frac{\omega^2}{c^2} \varepsilon \mu_x - k_x^2} \frac{\mu_x}{\mu_z}$  and  $k_{zD} = \sqrt{\frac{\omega^2}{c^2} \varepsilon_D \mu_D - k_x^2}$  denote the wave vector perpendicular to the interface in the HMM core and DPS cladding, respectively, and  $k_x$  is the propagation wave vector of the guided mode.

Similarly, the dispersion relation of the lowest-order guide modes for TE polarization in normal waveguide [Fig. 6(f)] is<sup>33</sup>

$$\sqrt{\frac{\omega^2}{c^2} \varepsilon_D \mu_D - k_x^2} d = 2 \arctan \left[ \frac{\mu_D \sqrt{k_x^2 - \frac{\omega^2}{c^2} \varepsilon_M \mu_M}}{\mu_M \sqrt{\frac{\omega^2}{c^2} \varepsilon_D \mu_D - k_x^2}} \right]. \quad (\text{S5})$$